# MTH 166

## Lecture-12

## Solution of Higher Order Homogeneous LDE with Constant Coefficients-II

**Topic:**

Solution of Higher order Homogeneous LDE with Constant coefficients

**Learning Outcomes:**

**1.** Formulation of $3^{rd}$ order and $4^{th}$ order homogeneous LDE when roots are given.

**2.** MCQ Practice.

**Formulation of LDE: $\boldsymbol{ay''' + by'' + cy' + dy = 0}$ when Roots are given:**

Let the three given roots be: $m_1, m_2$ and $m_3$.

Then required 3rd order homogeneous LDE is:

$y''' -$ (sum of roots taken one at a time ) $y'' +$ (sum of roots taken one at a time ) $y'$
$-$ (Product of roots)$y = 0$

i.e. $y''' - (m_1 + m_2 + m_3)y'' + (m_1m_2 + m_2m_3 + m_3m_1)y' - (m_1m_2m_3)y = 0$

or

Then required 3rd order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)y = 0$ where $D \equiv \frac{d}{dx}$

**Formulation of LDE: $\boldsymbol{ay^{iv} + by''' + cy'' + dy' + ey = 0}$ when Roots are given:**

Let the four given roots be: $m_1, m_2, m_3$ and $m_4$.

Then required 4th order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)(D - m_4)y = 0$ where $D \equiv \frac{d}{dx}$

**Find a homogeneous LDE with constant coefficients of lowest order which has the following particular solution:**

**Q 1.** $5 + e^{x} + 2e^{3x}$

**Sol.** Here: $5 + e^{x} + 2e^{3x} = 5e^{0x} + e^{1x} + 2e^{3x}$

So, $m_1 = 0, m_2 = 1, m_3 = 3$

Then required 3rd order homogeneous LDE is:

$y''' - (m_1 + m_2 + m_3)y'' + (m_1 m_2 + m_2 m_3 + m_3 m_1)y' - (m_1 m_2 m_3)y = 0$

$\Rightarrow y''' - (0 + 1 + 3)y'' + (0 + 3 + 0)y' - (0)y = 0$

$\Rightarrow y''' - 4y'' + 3y' = 0$ **Answer.**

**Q 2.** $xe^{-x} + e^{2x}$

**Sol.** Here: $xe^{-x} + e^{2x} = (0 + 1x)e^{-x} + e^{2x}$ $[(c_1 + c_2 x)e^{m_1 x} + c_3 e^{m_2 x}]$

So, $m_1 = -1, m_2 = -1, m_3 = 2$

Then required 3rd order homogeneous LDE is:

$$y''' - (m_1 + m_2 + m_3)y'' + (m_1 m_2 + m_2 m_3 + m_3 m_1)y' - (m_1 m_2 m_3)y = 0$$

$$\Rightarrow y''' - (-1 - 1 + 2)y'' + (1 - 2 - 2)y' - (2)y = 0$$

$\Rightarrow y''' - 3y' - 2y = 0$ **Answer.**

**Q 3.** $e^{-x} + \cos 5x + 3\sin 5x$

**Sol.** Here: $e^{-x} + \cos 5x + 3\sin 5x = c_1 e^{m_1 x} + e^{\alpha x}[c_2 \cos \beta x + c_3 \sin \beta x]$

So, $m_1 = -1, m_2 = 0 + 5i, m_3 = 0 - 5i$

Then required 3rd order homogeneous LDE is:

$y''' - (m_1 + m_2 + m_3)y'' + (m_1 m_2 + m_2 m_3 + m_3 m_1)y' - (m_1 m_2 m_3)y = 0$

$\Rightarrow y''' - (-1 + 5i - 5i)y'' + (-5i + 25 + 5i)y' - (25i^2)y = 0$

$\Rightarrow y''' + y'' + 25y' + 25y = 0$ **Answer.**

**Q 4.** $1 + x + e^x - 3e^{3x}$

**Sol.** Here: $1 + x + e^x - 3e^{3x} = (1 + x)e^{0x} + e^x - 3e^{3x}$

So, $m_1 = 0, m_2 = 0, m_3 = 1, m_4 = 3$

Then required 4th order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)(D - m_4)y = 0$ where $D \equiv \frac{d}{dx}$

$\Rightarrow (D - 0)(D - 0)(D - 1)(D - 3)y = 0$

$\Rightarrow D^2(D^2 - 4D + 3)y = 0$

$\Rightarrow (D^4 - 4D^3 + 3D^2)y = 0$

$\Rightarrow y^{iv} - 4y''' + 3y'' = 0$ **Answer.**

## MCQ Practice:

**Q1.** If $e^x, e^{4x}$ are solutions of differential equation $y'' + a(x)y' + b(x)y = 0$**,** then the values of $a(x)$ and b$(x)$ are:

**(A)** $a(x) = -5, b(x) = 4$

**(B)** $a(x) = 5, b(x) = 4$

**(C)** $a(x) = -5, b(x) = -4$

**(D)** $a(x) = 5, b(x) = -4$

**Sol.** Here $m_1 = 1, m_2 = 4$

Given equation is: $y'' + a(x)y' + b(x)y = 0$

Compare with: $y'' - (m_1 + m_2)y' + (m_1 m_2)y = 0$

$a(x) = -(m_1 + m_2) = -(1 + 4) = -5$ and $b(x) = (m_1 m_2) = 1(4) = 4$

Hence, Option (A) is right answer.

**Q2.** The intervals on which the differential equation $y' = 3\frac{y}{x}$ is normal are:

**(A)** $(-\infty, 0), (0, \infty)$ **(B)** $(-\infty, \infty)$

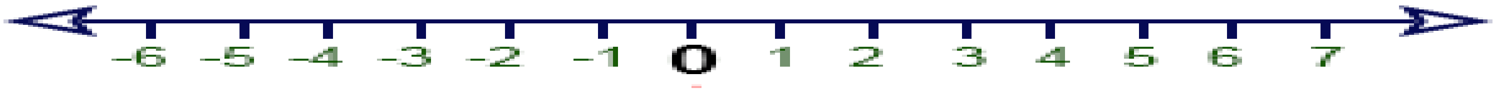
**(C)** $(-\infty, 1), (1, \infty)$ **(D)** None of these.

**Sol.** Given equation is: $xy' - 3y = 0$ (1)

Compare with: $a_0 y' + a_1 y = 0$

1. $\left.\begin{aligned} a_0 &= x \\ a_1 &= -3 \end{aligned}\right\}$ Being polynomials, $a_0$, $a_1$ are continuous on $(-\infty, \infty)$

2. $a_0 \neq 0 \qquad \Rightarrow x \neq 0$




Equation (1) is normal on subintervals: $(-\infty, 0), (0, \infty)$

Hence, Option (A) is right answer.

**Q3.** If $y = e^{at}$ is solution of $y'' - 5y' + 4y = 0$, then possible value of $a$ is:

**(A)** $a = 2$ **(B)** $a = 3$

**(C)** $a = 4$ **(D)** $a = 5$

**Sol.** Given equation is: $y'' - 5y' + 4y = 0$ (1)

S.F. : $(D^2 - 5D + 4)y = 0$ where $D \equiv \frac{d}{dx}$

A.E. : $(D^2 - 5D + 4) = 0$ $\Rightarrow (D - 4)(D - 1) = 0$

$\Rightarrow D = 4{,}1$ (real and unequal roots)

Let $m_1 = 4$ and $m_2 = 1$

$\therefore$ General Solution of equ. (1) is: $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$\Rightarrow y = c_1 e^{4x} + c_2 e^{1x}$ $\Rightarrow a = 4$ or $a = 1$

Hence, Option (C) is right answer.

**Q4.** The general solution of $y'' - 9y = 0$ is:

**(A)** $Ae^{-3x} + Be^{3x}$ **(B)** $Ae^{3x} + Be^{3x}$

**(C)** $Ae^{-3x} + Be^{-3x}$ **(D)** None of these

**Sol.** The given equation is: $y'' - 9y = 0$ (1)

S.F. : $(D^2 - 9)y = 0$ where $D \equiv \frac{d}{dx}$

A.E. : $(D^2 - 9) = 0$ $\Rightarrow D^2 = 9$ $\Rightarrow D = \pm 3$ (real and unequal roots)

Let $m_1 = -3$ and $m_2 = 3$

∴ General Solution of equation (1) is given by:

$y = Ae^{m_1 x} + Be^{m_2 x}$ $\Rightarrow y = Ae^{-3x} + Be^{3x}$

Hence, Option (A) is right answer.

**Q5.** The general solution of $y'' + 4y = 0$ is:

**(A)** $(Acos2x + Bsin\ 2x)$ **(B)** $Ae^{2x} + Be^{-2x}$

**(C)** $(A + Bx)e^{-2x}$ **(D)** None of these

**Sol.** The given equation is: $y'' + 4y = 0$ (1)

S.F. : $(D^2 + 4)y = 0$ where $D \equiv \frac{d}{dx}$

A.E. : $(D^2 + 4) = 0$ $\Rightarrow D^2 = -4$ $\Rightarrow D = \pm 2i$ (complex roots)

Let $m_1 = 0 + 2i$ and $m_2 = 0 - 2i$

$\therefore$ General Solution of equation (1) is given by:

$y = e^{0x}(Acos2x + Bsin\ 2x) \Rightarrow y = (Acos2x + Bsin\ 2x)$

Hence, Option (A) is right answer.

**Q6.** The Wronskian of functions: $(1, \sin x, \cos x)$is:

**(A)** 0 **(B)** 1

**(C)** $-1$ **(D)** None of these

**Sol.** Let $(y_1, y_2, y_3) = (1, \sin x, \cos x)$

Wronskian, $W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} 1 & \sin x & \cos x \\ 0 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \end{vmatrix}$

Expanding by first column:

$W = 1(-cos^2x - sin^2x) - 0 + 0 = -1(cos^2x + sin^2x) = -1$

Hence, Option (C) is right answer.

**Q7.** The general solution of $y'' - 10y' + 25y = 0$ is:

**(A)** $Ae^{4x} + Be^{5x}$ **(B)** $Ae^{5x} + Be^{5x}$

**(C)** $Ae^{4x} + Be^{7x}$ **(D)** $Ae^{5x} + Bxe^{5x}$

**Sol.** Given equation is: $y'' - 10y' + 25y = 0$ (1)

<u>S.F. :</u> $(D^2 - 10D + 25)y = 0$ where $D \equiv \frac{d}{dx}$

<u>A.E. :</u> $(D^2 - 10D + 25) = 0 \quad \Rightarrow (D-5)^2 = 0$

$\Rightarrow D = 5, 5$ (real and equal roots)

Let $m_1 = 5$ and $m_2 = 5$

$\therefore$ General Solution of equation (1) is given by:

$y = (A + Bx)e^{m_1 x} \quad \Rightarrow y = (A + Bx)e^{5x} \quad \Rightarrow y = Ae^{5x} + Bxe^{5x}$

Hence, Option (D) is right answer.

Do More Practice of MCQ and identify the areas which need more attention

8 The value of b in the differential equation $ay'' + by' + cy = 0$ for which $e^{-2x}, e^{3x}$ are the fundamental solutions

(a) 1 (b) -1 c) 6 (d) -6

9 General solution of the LDE y''-25y = 0 is

(a) $Ae^{-25x} + Be^{25x}$ (b) $(A + Bx)\,e^{5x}$ (c) $Ae^{-5x} + Be^{5x}$ (d) $Ae^{5x} + Bxe^{-5x}$

10 If $e^{kx}$ is solution of $4y'' + 7y' + 3y = 0$ then $k$ is

a) 1 b) 2 c) ¾ d) $-3/4$

11 If $1 + 2\sinh 2x + 3\cosh 2x$ be particular solution of homogeneous equation then its lowest order is

a) 2 b) 3 c) 4 d) 5

12 The Wronskian of the functions $1, \cos x, \sin x$ is

(a) 1 (b) 0 (c) 2 (d) None of these

13 Which of the following equation have $e^{-i3x}, e^{i3x}$ as fundamental solutions?

a) $y'' + 6y' + 9y = 0$ (b) $y'' - 6y' + y = 0$ (c) $y'' + 9y = 0$ d) $y'' - 9y = 0$

14 The set of linearly independent solutions of the equation $y^{iv} + 8y'' - 9y = 0$

(a) $\{e^{x}, e^{-2x}, \sin 3x, \cos 3x\}$ (b) $\{xe^{x}, e^{-2x}, \sin 3x, \cos 3x\}$

(c) $\{xe^{-2x}, \sin 3x, \cos 3x\}$ (d) $\{e^{x}, e^{-x}, \sin 3x, \cos 3x\}$

15 The set of linearly independent solutions of the differential equation $y^{iv} + y''' + 14y'' + 16y' - 32y = 0$

(a) $\{e^{x}, e^{-2x}, \sin 4x, \cos 4x\}$ (b) $\{xe^{x}, e^{-2x}, \sin 4x, \cos 4x\}$

(c) $\{xe^{-2x}, \sin 4x, \cos 4x\}$ (d) $\{e^{x}, e^{-2x}, x\sin 4x, x\cos 4x\}$

16 On which interval the given differential equation $x(1-x)y'' - 3xy' - y = 0$ is normal

(a) $(-\infty, 0)$ (b) $(0,1)$ (c) $(1, \infty)$ (d) All of above

17 The non- trivial solution of the boundary value problem $y'' + w^2 y = 0$ satisfing the conditions $y(0) = 0$ and $y(\pi) = 0$ and for any integer n is given by

(a) $y = a\cos nx$ (b) $y = a\sin nx$ (c) $y = a\cos nx + b\sin nx$ (d) 0

18 .Classify the given differential given equation: $x^3 y''' + 9x^2 y'' + 18xy' + 6y = 0$

(a)Above equation is Higher order homogeneous , Non linear differential equation with constant coefficients

(b)Above equation is Higher order homogeneous linear differential equation with variable coefficients

(c)Above equation is Higher order Non homogeneous linear differential equation with constant coefficients

(d)Above equation is Higher order Non homogeneous , Non linear differential equation with variable coefficients

19 .he solution of the differential equation $y'' + y = 0$ satisfing the conditions $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = 2$

(a) $\cos x + 2\sin x$ (b) $\cos x + \sin x$ (c)$2\cos x + \sin x$ (d)$2\cos x + 2\sin x$

20 The general solution of differential equation $y'' - 4y = 0$ is

(a) $Ae^{2x} + Be^{-2x}$ (b) $A\cos 2x + B\sin 2x$ (c) $(A + Bx)e^{2x}$ (d) $(A + Bx)\sin 2x$

21 The differential equation whose two linear independent solutions are $e^{-x}, e^{x}$ is

(a) $y' - y = 0$ (b) $y'' - y = 0$ (c) $y'' - 2y' + y = 0$ (d) $y'' + y = 0$

22 $y(x) = e^{-x}\left(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right) + c_3e^{2x}$ is general solution of

(a) $y''' + 4y = 0$ (b) $y''' - 8y = 0$ (c) $y''' + 8y = 0$ (d) $y''' - 2y'' + y' - 2y = 0$

23 Let $f_1 = x, f_2 = x^4, f_3 = 1 + x, f_4 = 1$, then Wronskian $W(f_1, f_2, f_3, f_4) =$

(a) $6x$ (b) 2 (c) $3 - 4x$ (d) 0

24 If $y = e^{4t}$ is solution of $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + ky = 0$, then value of $k$ is

(a) 2 (b) 3 (c) 4 (d) 5

25 Consider the differential equation $\frac{d^2y}{dx^2} - 7y = 0$, which of the following is correct?

(a) The roots of the auxiliary equation are 0 and 7.

(b) There is no auxiliary equation for a differential equation of this type.

(c) The auxiliary equation has a repeated root of $\sqrt{7}$.

(d) The roots of the auxiliary equation $\sqrt{7}$ and $-\sqrt{7}$.

26 If $y(x) = Ae^{3x} + Be^{-2x} + Ce^{x}$ is a solution of $y''' - 2y'' - 5y' + 6y = 0$, $y(0) = 0$ then which of the following is

(a) A +B +C = 0 (b) A + B - C = 0 (c) A + B + C = 0 (d) None of these

27 If differential equation $y' = \frac{3y}{x}$ is normal over the interval I then I is

(a) $(-\infty, 2)$ (b) $(-\infty, \infty)$ (c) $(-2, \infty)$ (d) $(-\infty, 0)$

28 . What is lowest order of homogeneous linear homogeneous differential equation with constant coefficient whose particular solution is $3\cos 2x + 5\sinh 3x$

(a) 2 (b) 3 (c) 4 (d) 6

29 What are the characteristic roots of a homogeneous linear DE with constant coefficients having $4 + xe^{2x}$ as its particular solution?

(a) 0, 2 (b) 4, 2 (c) 4, 2, 2 (d) 0, 2, 2

30 The functions $f_1, f_2, \_\_\_\_\_\_ f_n$ are said to be linearly dependent if $W(f_1, f_2, \_\_\_\_ f_n)$ is

(a) 0 (b) $\neq 0$ (c) 1 (d) None of these

31 Particular solution of the problem

$(D^2 + 4)y = 0$ ; $y(0) = 0$, $y'(\pi) = 2$ is

(a) y(x) = sin 2x (b) y(x) = cos2x (c) y(x) = tan2x (d) none of these

32 If the roots of the auxiliary equation of a linear differential equation are 4,4 then its complementary function is

(a) $C_1e^{4x} + C_2e^{4x}$ (b) $(C_1 + C_2x)e^{4x}$ (c) $(C_1 + C_2x^2)e^{4x}$ (d) $(C_1x + C_2x^2)e^{4x}$

Thank You
Have a nice day